

ASSIGNMENT - 1

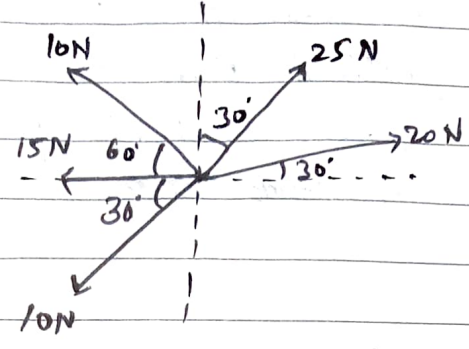
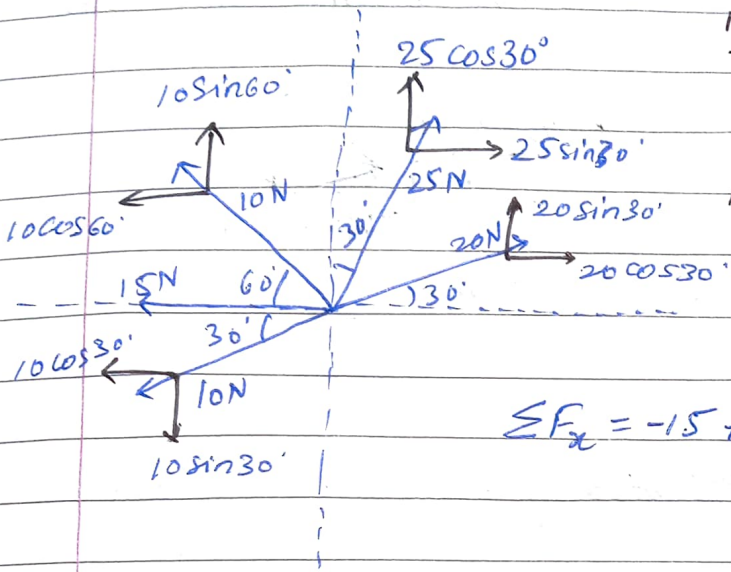
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IIoT B1

Application No. : 131220028552



Q1. Find the Resultant for the given force system. Also find the direction and position.



$$\Sigma F_x = -15 + 20 \cos 30^\circ + 25 \sin 30^\circ - 10 \cos 30^\circ - 10 \cos 60^\circ$$

$$\Sigma F_x = 1.16 \text{ N} \quad \text{--- (1)}$$

and $\Sigma F_y = 25 \cos 30^\circ + 20 \sin 30^\circ + 10 \sin 60^\circ - 10 \sin 30^\circ$

$$\Sigma F_y = 25(0.86) + 20(0.5) + 10\left(\frac{\sqrt{3}}{2}\right) - 10(0.5)$$

$$\Sigma F_y = 35.31 \text{ N} \quad \text{--- (2)}$$

from (1) & (2)

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

$$R = \sqrt{(1.16)^2 + (35.31)^2}$$

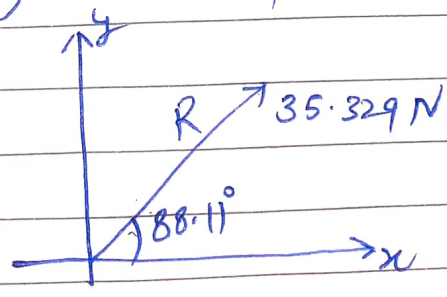
$$R = 35.329 \text{ N}$$

$$\theta = \tan^{-1} \left| \frac{\Sigma F_y}{\Sigma F_x} \right|$$

$$= \tan^{-1} \left| \frac{35.31}{1.16} \right|$$

$$\theta = 88.11^\circ$$

Since ΣF_x & ΣF_y both are (+ve) so Resultant lies in 1st quadrant.



Q.2. Two forces of 100N and 150N are acting simultaneously at a point. What is the resultant of these two forces, if the angle between them is 45° ?

Solⁿ \Rightarrow Given: $F_1 = 100\text{N}$

$$F_2 = 150\text{N}$$

$$\text{Angle b/w } F_1 \text{ \& } F_2 (\theta) = 45^\circ$$

$$\therefore R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \theta}$$

$$= \sqrt{(100)^2 + (150)^2 + 2 \times 100 \times 150 \cos 45^\circ}$$

$$= \sqrt{10000 + 22500 + (30000 \times 0.707)}$$

$$= 232\text{ N}$$

Ans

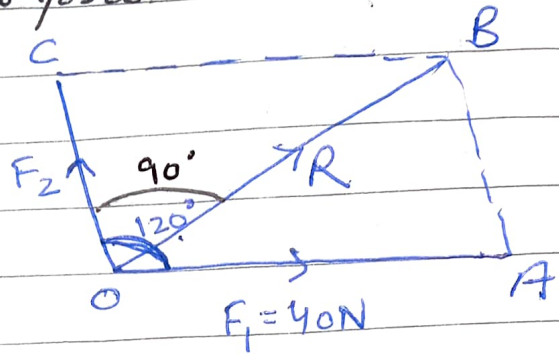
Q3: Two forces act at an angle of 120° . The bigger force is of 40N and the resultant is \perp to smaller one. find the smaller force.

Solⁿ \Rightarrow Given:

Angle b/w two forces = $\angle AOC = 120^\circ$

$$F_1 = 40\text{N}$$

Angle b/w Resultant and small force = 90°



from geometry $\angle AOB = \alpha = 120 - 90 = 30^\circ$

also we know that $\tan \alpha = \frac{F_2 \sin \theta}{F_1 + F_2 \cos \theta}$

$$\tan 30^\circ = \frac{F_2 \sin 120^\circ}{40 + F_2 \cos 120^\circ}$$

$$\tan 30^\circ = \frac{F_2 \sin 60^\circ}{40 + F_2 (-\cos 60^\circ)}$$

$$0.577 = \frac{F_2 \times 0.866}{40 - F_2 \times 0.5} = \frac{0.866 F_2}{40 - 0.5 F_2}$$

$$40 - 0.5 F_2 = \frac{0.866 F_2}{0.577} = 1.5 F_2$$

$$2 F_2 = 40$$

$$\Rightarrow F_2 = 20$$

Ans

Q4 Find the magnitude of the two forces, such that if they act at right angles, their resultant is $\sqrt{10}$ N. But if they act at 60° , their resultant is $\sqrt{13}$ N.

Soln:

Case I: Forces acting at right angles

$$R = \sqrt{F_1^2 + F_2^2}$$

$$\sqrt{10} = \sqrt{F_1^2 + F_2^2}$$

Sq. both side:

$$10 = F_1^2 + F_2^2 \quad \text{--- (1)}$$

Case II: force acting at 60° angle

$$\sqrt{13} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos 60^\circ}$$

$$13 = F_1^2 + F_2^2 + 2F_1F_2 \times 0.5$$

$$-F_1F_2 = F_1^2 + F_2^2 - 13$$

$$-F_1F_2 = 10 - 13 \quad (\text{from (1)})$$

$$F_1F_2 = 3$$

now,

$$(F_1 + F_2)^2 = F_1^2 + F_2^2 + 2F_1F_2 = 10 + 6 = 16$$

$$\therefore F_1 + F_2 = \sqrt{16} = 4 \quad \text{--- (2)}$$

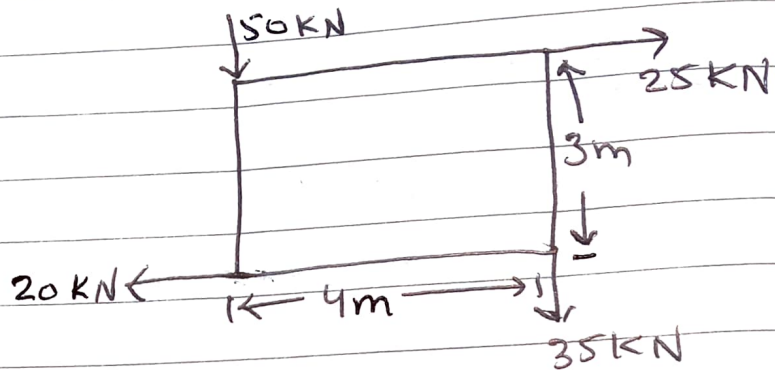
$$\text{Similarly, } (F_1 - F_2)^2 = F_1^2 + F_2^2 - 2F_1F_2 = 10 - 6 = 4$$

$$\therefore F_1 - F_2 = \sqrt{4} = 2 \quad \text{--- (3)}$$

Solving eq (2) & (3),

$$F_1 = 3\text{N and } F_2 = 1\text{N.}$$

Q5 A system of forces are acting at the corners of a rectangular block as shown in fig. 2.4. Determine the magnitude and direction of the resultant force.



Soln:

$$\begin{aligned}\sum F_x &= 25 - 20 \\ &= 5 \text{ kN}\end{aligned}$$

$$\sum F_y = -50 + (-35) = -85 \text{ N}$$

\therefore magnitude of Resultant force

$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2} = \sqrt{(5)^2 + (-85)^2}$$

$$R = 85.15 \text{ kN}$$

$$\text{now, } \tan \theta = \frac{\sum F_y}{\sum F_x} = \frac{-85}{5} = -17$$

$$\theta = 86.6^\circ$$

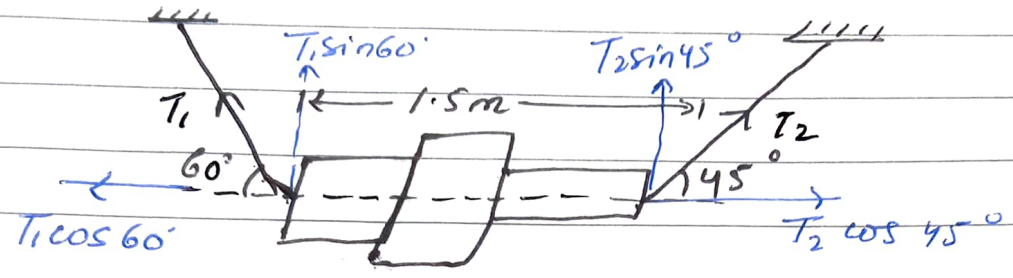
Since $\sum F_x$ is positive & $\sum F_y$ is negative,
 \therefore resultant lies b/w 270° & 360° , Thus
 actual angle of the resultant force

$$\theta = 360^\circ - 86.6^\circ$$

$$\theta = 273.4^\circ$$

Ans

Q6. A machine component 1.5m long and weight 1000N is supported by two ropes AB and CD as shown in fig given below. Calculate the tensions T_1 and T_2 in the ropes AB and CD.



Solⁿ ⇒ Given: Weight of component = 1000N

→ Horizontal components of T_1 & T_2 are equal

$$\therefore T_1 \cos 60 = T_2 \cos 45$$

$$T_1 = \frac{\cos 45}{\cos 60} \times T_2 = \frac{0.707}{0.5} \times T_2 = 1.414 T_2$$

→ Vertical components:

$$T_1 \sin 60 + T_2 \sin 45 = 1000$$

$$(1.414 T_2) \times 0.866 + T_2 \times 0.707 = 1000$$

$$1.93 T_2 = 1000$$

$$\therefore T_2 = \frac{1000}{1.93} = 518.1 \text{ N}$$

$$\text{and } T_1 = 1.414 \times 518.1 = 732.6 \text{ N}$$

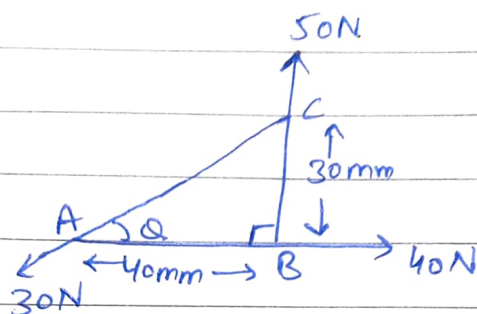
Q7. A ΔABC has its side $AB = 40\text{mm}$ along $+x$ -axis and side $BC = 30\text{mm}$ along $+y$ -axis. Three forces of 40N , 50N and 30N act along the sides AB , BC and CA respectively. Determine magnitude of the resultant of such a system of forces.

using pythagoras theorem:

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = (40)^2 + (30)^2$$

$$AC = 50\text{mm}$$



$$\therefore \sin \theta = \frac{30}{50} = 0.6$$

$$\cos \theta = \frac{40}{50} = 0.8$$

Resolving all the forces horizontally (i.e. along AB),

$$\begin{aligned} \sum F_x &= 40 - 30 \cos \theta \\ &= 40 - (30 \times 0.8) = 16\text{N} \end{aligned}$$

now resolving all the forces vertically (i.e. along BC)

$$\begin{aligned} \sum F_y &= 50 - 30 \sin \theta \\ &= 50 - (30 \times 0.6) = 32\text{N} \end{aligned}$$

$$\begin{aligned} \text{now, } R &= \sqrt{(\sum F_x)^2 + (\sum F_y)^2} \\ &= \sqrt{(16)^2 + (32)^2} \end{aligned}$$

$$R = 35.8\text{N} \quad \text{Ans}$$

Q8. The forces 20N, 30N, 40N, 50N and 60N are acting at one of the angular points of a regular hexagon, towards the other five angular points, taken in order. Find the magnitude and direction of the resultant force.

$$\begin{aligned} \Sigma f_x &= 20 \cos 0^\circ + 30 \cos 30^\circ \\ &\quad + 40 \cos 60^\circ + 50 \cos 90^\circ \\ &\quad + 60 \cos 120^\circ \\ &= \cancel{200} (20 \times 1) + (30 \times 0.866) \\ &\quad + (40 \times 0.5) + (50 \times 0) + 60(-0.5) \\ &= 36.0 \text{ N} \quad \text{--- (1)} \end{aligned}$$

$$\begin{aligned} \Sigma f_y &= 20 \sin 0^\circ + 30 \sin 30^\circ + 40 \sin 60^\circ + 50 \sin 90^\circ + 60 \sin 120^\circ \\ &= (20 \times 0) + (30 \times 0.5) + (40 \times 0.866) + (50 \times 1) \\ &\quad + (60 \times 0.866) \\ &= 151.6 \text{ N} \quad \text{--- (2)} \end{aligned}$$

$$R = \sqrt{\Sigma f_x^2 + \Sigma f_y^2} = \sqrt{(36)^2 + (151.6)^2}$$

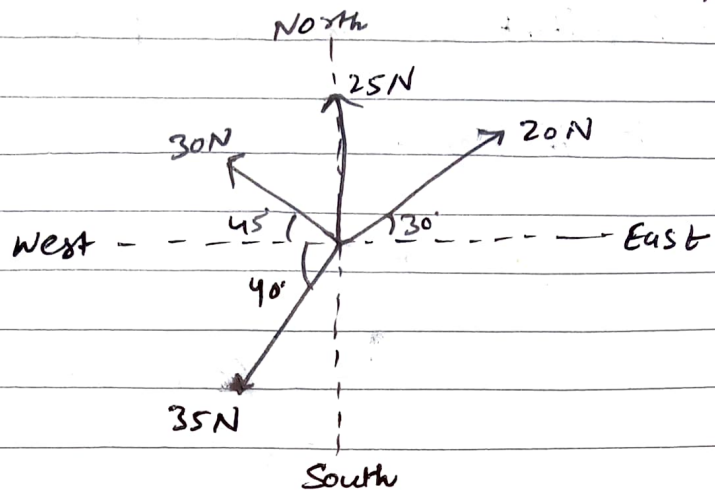
$$R = 155.8 \text{ N}$$

also, $\tan \alpha = \frac{\Sigma f_y}{\Sigma f_x} = \frac{151.6}{36.0}$

$$\alpha = \tan^{-1}(4.211)$$

$$\alpha = 76.6^\circ$$

Q9: find the magnitude and direction of the resultant force



$$\begin{aligned}\sum f_x &= 20 \cos 30^\circ + 25 \cos 90^\circ + 30 \cos 135^\circ + 35 \cos 220^\circ \\ &= (20 \times 0.866) + (25 \times 0) + 30(-0.707) + 35(-0.766) \\ &= -30.7 \text{ N}\end{aligned}$$

$$\begin{aligned}\sum f_y &= 20 \sin 30^\circ + 25 \sin 90^\circ + 30 \sin 135^\circ + 35 \sin 220^\circ \text{ N} \\ &= (20 \times 0.5) + (25 \times 1.0) + (30 \times 0.707) + 35(-0.6428) \\ &= 33.7 \text{ N}\end{aligned}$$

$$R = \sqrt{\sum f_x^2 + \sum f_y^2} = \sqrt{(-30.7)^2 + (33.7)^2}$$

$$R = 45.6 \text{ N}$$

$$\tan \theta = \frac{\sum f_y}{\sum f_x} = \frac{33.7}{-30.7} = -1.098$$

$$\theta = \tan^{-1}(-1.098)$$

$$\theta = 47.7^\circ$$

but as $\sum f_x$ is negative & $\sum f_y$ is positive, \therefore Resultant lies in 2nd quadrant.

$$\theta = 180^\circ - 47^\circ = 133.3^\circ \text{ Ans}$$

Q10. Find the angle between two equal forces P , when their resultant is equal to (i) P and (ii) $P/2$.

(i) if Resultant is equal to P .

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \alpha}$$

$$P = \sqrt{P^2 + P^2 + 2 \cdot P \cdot P \cos \alpha}$$

$$= P \sqrt{2 + 2 \cos \alpha}$$

$$\therefore 2 \cos \alpha = -1 \text{ or } \cos \alpha = -0.5 \text{ or } \alpha = 120^\circ$$

(ii) when resultant is equal to $P/2$, then

$$0.5P = \sqrt{P^2 + P^2 + 2P \cdot P \cos \alpha}$$

$$= P \sqrt{2 + 2 \cos \alpha}$$

$$2 \cos \alpha = -1.75 \text{ or } \cos \alpha = -0.875$$

$$\therefore \alpha = 151^\circ \text{ Ans}$$